

Maths Calculation Policy

For the New National Curriculum

Always think:

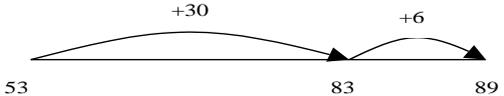
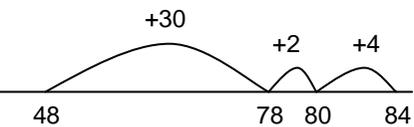
Can I do it mentally?

Can I do it with a jotting?

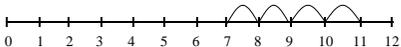
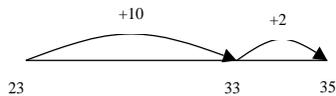
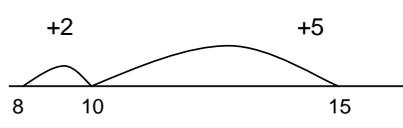
Do I need a written method (vertical layout)?

Do I need a calculator?

ADDITION GUIDELINES

Stage One	Stage Two	Stage Three
<p>Prerequisite skills (based on the practical) Counting numbers to 20</p>  <p>(using familiar / practical resources)</p> <p>Place numbers to 20 in order</p> <p>Bonds up to 10 and to make 10</p>  <p>1 more than a number</p>   <p>Addition as combining groups</p>  <p>1, 2, 3, 4 1, 2, 3</p> <p style="margin-left: 40px;">1, 2, 3, 4, 5, 6, 7</p> <p>Addition as counting on</p>  <p>Doubling numbers within 20</p> 	<p>Prerequisite skills (based on the practical)</p> <p>Relate number bonds to 10 to add multiples of 10 up to a total of 100 e.g. if 3 + 4 is 7 then 30 + 40 is 70</p>  <p>Use familiar objects to recognise the place value of 2 digit numbers.</p>  <p>Recognise and explain 24 is '2 tens and 4 ones'</p>   <p>Progressing to: PARTITIONING AND RECOMBINING Partition into tens and ones and recombine Pre J10 (before jumping in 10s)</p> $12 + 23 = 10 + 2 + 20 + 3$ $= 30 + 5$ $= 35$ <p>Model this on a bead bar and practise on 100-beadstrings, showing the 'collection' of 10s and then the ones. i.e. "2 tens and 1 ten makes 3 tens, which is 30. Then 3 and 2 makes 5 ones. Altogether we can see 3 tens and 5 ones, which is 35." Check by counting in tens and ones along the bead bar. Model and practise with place value arrow cards, numicon, bead strings or Dienes, using known facts and place value to calculate each step.</p>	<p>Partition into tens and ones</p> <ul style="list-style-type: none"> Partition one number and recombine. Count on by partitioning the second number only <p>e.g.</p> $36 + 53 = 53 + 30 + 6$ $= 83 + 6$ $= 89$ <p>As modelled below as necessary</p>  <p>Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.</p> $48 + 36 = 84$  <p>First J10 then T10</p> <p>Add a near multiple of 10 to a two-digit number (Overjumping - OJ)</p> <p>Secure mental methods by using a number line to model the method. Continue as in Stage 2 but with appropriate numbers</p> <p>E.g. 35 + 19 is the same as 35 + 20 - 1.</p> <p>Once a child is able to add 3 digit numbers on a number line securely move on to vertical expansion.</p> <p>+ / = signs and missing numbers</p> <p>Continue using a range of equations as in Stage 1 and 2 but with appropriate, larger numbers.</p>

ADDITION GUIDELINES

Stage One	Stage Two	Stage Three
<p>Number bonds to 10</p> <div style="text-align: center;">  </div> <p><u>+ / = signs and missing numbers</u></p> <p>Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.</p> <p>2 = 1 + 1 2 + 3 = 4 + 1 3 = 3 2 + 2 + 2 = 4 + 2</p> <p>Missing numbers need to be placed in all possible places.</p> <p>3 + 4 = □ □ = 3 + 4 3 + □ = 7 7 = □ + 4 □ + 4 = 7 7 = 3 + □ □ + ▽ = 7 7 = □ + ▽</p> <p><u>The Number Line</u></p> <p>Children use a numbered line to count on in ones. Children use number lines and practical resources to support calculation and teachers <i>model</i> the use of the number line.</p> <p>e.g. 7 + 4:</p> <div style="text-align: center;">  </div> <p><u>Number line Teaching Points:</u> <i>Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation.</i> <i>Numbers ('landmarks') are written below the line.</i> <i>Size of the 'jumps' are written above the 'jumps'.</i></p>	<p><u>Count on in tens and ones</u> <u>J10</u> (jumping in 10s)</p> <p>23 + 12 = 23 + 10 + 2 = 33 + 2 = 35</p> <p><i>Model this on a number line starting at 23 and jumping 10 (J10) to make 33 and then add 2 in one jump.</i></p> <div style="text-align: center;">  </div> <p><u>The Empty Number Line:</u> <u>T10</u> (Targeting the 10, partitioning and bridging through 10)</p> <p>The steps in addition often bridge through a multiple of 10 e.g.</p> <p>Children should be able to partition the 7 to relate adding the 2 first to target the 10 and then the 5.</p> <p>8 + 7 = 15</p> <div style="text-align: center;">  </div> <p><i>O10 not recommended for written methods but can be used as a strategy in mental methods.</i></p> <p><u>+ / = signs and missing numbers</u></p> <p>Continue using a range of equations as in Stage 1 but with appropriate, larger numbers.</p> <p>Extend to 14 + 5 = 10 + □ and 32 + □ + □ = 100 35 = 1 + □ + 5</p>	

ADDITION GUIDELINES

Stage Four	Stage Five	Stage Six
<p><u>Partition into hundreds, tens and ones and recombine</u> Either partition both numbers and recombine or partition the second number only e.g. $358 + 73 = 358 + 70 + 3$ $= 428 + 3$ $= 431$</p> <div style="text-align: center;"> </div> <p><u>Horizontal Expansion</u></p> $367 + 185 = 552$ $\begin{array}{r} 367 \\ +185 \\ \hline 400 \text{ (300+100)} \\ 140 \text{ (60+80)} \\ \underline{12} \text{ (7+5)} \\ 552 \end{array}$ <p>Moving on to</p> $367 + 185 = 552$ $\begin{array}{r} 367 \\ +185 \\ \hline 400 \\ 140 \\ \underline{12} \\ 552 \end{array} \quad \text{(without use of brackets)}$	<p><u>Adding the least significant digits first</u></p> $\begin{array}{r} 247 \\ + 176 \\ \hline 13 \text{ (7+6)} \\ 110 \text{ (40 + 70)} \\ \underline{300} \text{ (200 + 100)} \\ 423 \end{array}$ <p>Working from <u>left to right</u>: 'Read' the answer from left to right, using knowledge of place value and referring to the value of each digit i.e.: "four hundred and twenty three" <u>NOT</u> adding up columns for the final answer</p> <p>Moving on to</p> $\begin{array}{r} 247 \\ + 376 \\ \hline 13 \\ 110 \\ \underline{500} \\ 623 \end{array} \quad \text{(without use of brackets)}$ <p>Moving on to a compact method</p> $\begin{array}{r} 247 \\ + 376 \\ \hline \underline{623} \\ 11 \end{array}$ <p>Working from <u>right to left</u>: "7 + 6 is 13. Partition the 13 into 10 and 3, 'carry' the ten into the tens column, writing it as 1 to represent one ten." n.b. the '1' can be written at the top or bottom of the calculation.</p> <p>It is <u>NOT</u> "carry the 1"</p> <p><i>Consolidation and practice of the previous key facts.</i></p>	<p><u>Extend to numbers with at least four digits</u></p> $3587 + 675 = 4262$ $\begin{array}{r} 3587 \\ + 675 \\ \hline \underline{4262} \\ 111 \end{array}$ <p>Revert to expanded methods if the children experience any difficulty.</p> <p><u>Partition into hundreds, tens, ones and decimal fractions and recombine</u> Either partition both numbers and recombine or partition the second number only e.g. $35.8 + 7.3 = 35.8 + 7 + 0.3$ $= 42.8 + 0.3$ $= 43.1$</p> <div style="text-align: center;"> </div> <p>Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).</p> $\begin{array}{r} 72.8 \\ + 54.6 \\ \hline \underline{127.4} \\ 1 \quad 1 \end{array}$

ADDITION GUIDELINES		
Stage Four	Stage Five	Stage Six
<p>Why most significant digit first and then least significant digit first? When adding and subtracting on a number line we start with the most significant digit first (e.g. add the tens then add the units). This is why horizontal expansion starts with the most significant digit first. Once the children are secure in this, it changes to adding the least significant digit first. This bridges the gap between these two stages (many children will only need to see it a few times to understand the relationship but others may need more experience at each stage)</p> <p><i>It is crucial to know or be able to derive key number facts TO + TO mentally or with jottings before progressing to Stage Five.</i></p> <p><u>+ / = signs and missing numbers</u> Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.</p> <p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>	<p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>	<p><u>Extend to numbers with more than 4 digits or decimals with up to 3 places</u> $13.86 + 9.481 = 23.341$</p> $\begin{array}{r} 13.86 \\ + 9.481 \\ \hline 23.341 \\ \small{111} \end{array}$ <p>$12350 + 4921$</p> $\begin{array}{r} 12350 \\ + 4921 \\ \hline 17271 \\ \small{1} \end{array}$ <p>Revert to expanded methods if the children experience any difficulty.</p> <p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>

End of Year Objectives for Addition

Yr1 – recall and jottings for $O+O$, $T+O$, $T+T$, $TO+O$ (within 20 including 0)

Yr2 – $TO+O$, $T+TO$, $TO+TO$, $O+O+O$

Yr3 – mental methods for $HTO + O$, $HTO+T$, $HTO+H$; written methods for $HTO+TO$, $HTO+HTO$

Yr4 – written methods as above and $ThHTO + ThHTO$, $O.t+O.t$, $£O.th+£O.th$

Yr5 – written method for addition of numbers with more than four digits; 2 or more integers, decimals with 2dp e.g. $29.78 + 54.34$

Yr6 – As above

Differentiation Steps for each Stage:

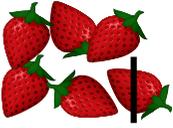
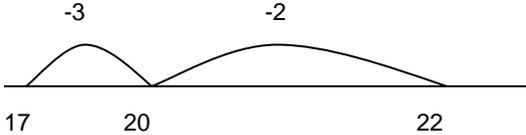
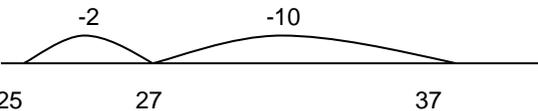
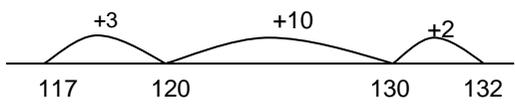
- Not crossing tens
- Crossing Tens
- Crossing Hundreds Only
- Crossing Tens and Hundreds

In addition:

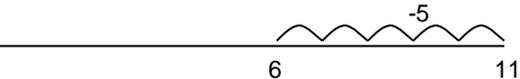
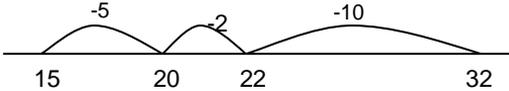
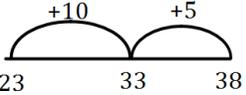
- The number line must be modelled as an image to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation can be carried out mentally then do not give it to practise vertical calculation, e.g. $TO + TO$ should not be calculated vertically.

Always present calculations horizontally in order to consider mental calculations first.

SUBTRACTION GUIDELINES

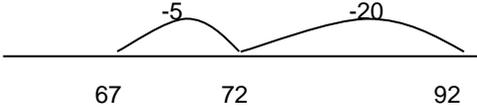
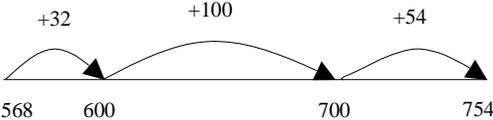
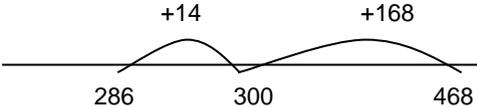
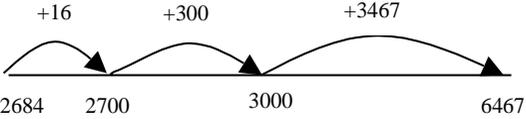
Stage One	Stage Two	Stage Three
<p>Prerequisite skills (based on the practical) Number bonds to 10</p>  <p>Counting back from 20</p> <p>Find one less than a given number</p>  <p>Subtract using quantities and objects 2 single digit numbers</p>  <p>Count back to subtract single digit numbers</p>  <p>There are two concepts linked to subtraction: Subtract - where it is natural to count back to 'take away' Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].</p> <p>Understand subtraction as 'take away'</p> 	<p>There are two concepts linked to subtraction: Subtract - where it is natural to count back to 'take away' Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].</p> <p>Use known number facts and place value to subtract</p> <p>Using knowledge of number bonds to subtract mentally from multiples of 10s e.g. $30 - 4$</p> <p>Using knowledge of number bonds to subtract mentally multiples of 10 from multiples of 10 e.g. if $7 - 4 = 3$ then $70 - 40 = 30$</p> <p>Using knowledge of number bonds to subtract mentally e.g. if $8 - 3 = 5$ then $28 - 3 = 25$</p> <p>Use of T10 for TO-O $22 - 5 = 22 - 2 - 3$ $= 20 - 3$</p>  <p>Use of J10 using multiples of 10 Example: $80 - 30$</p> <p>Use of J10 $37 - 12 = 37 - 10 - 2$ $= 27 - 2$ $= 25$</p> 	<p>Use known number facts and place value to subtract Continue as in Stage 2 but with appropriate numbers e.g. $197 - 53 = 144$</p> <p style="text-align: center;">-3 -50</p>  <p>Use counting on to find the difference $132 - 117 = 15$</p>  <p>Secure knowledge in use of J10 and T10 to count back and find the difference.</p> <p>TO-TO, HTO-TO, HTO-HTO</p> <p>By the end of this stage children should know complements to 100. They can then use this knowledge to calculate HTO-TO, HTO-HTO.</p> <p>Subtract mentally a 'near multiple of 10' to or from a two-digit number Continue as in Stage 2 but with appropriate numbers e.g. $78 - 49$ is the same as $78 - 50 + 1$</p> <p>- = signs and missing numbers (inverse)</p> <p>Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.</p>

SUBTRACTION GUIDELINES

Stage One	Stage Two	Stage Three								
<p>Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.</p> <p>I have 11 toy cars. I lost 5 of them. How many are left?</p> <p>Start with bead strings / bars and move onto number lines below.</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Use the vocabulary related to subtraction and symbols to describe and record subtraction number sentences (for the example above it would be $11 - 5 = 6$)</p> <p>Recording by</p> <ul style="list-style-type: none"> - drawing jumps on prepared lines / tracks <p>Use practical resources to find the difference between two small numbers e.g. 6 and 7</p> <div style="margin: 10px 0;">  </div> <p>Count on from smallest to largest number to find the difference where numbers are close in value. (e.g. $9-7$)</p> <p><u>- = signs and missing numbers(inverse)</u></p> <table style="width: 100%; border: none;"> <tr> <td style="padding-right: 20px;">$7 - 3 = \square$</td> <td>$\square = 7 - 3$</td> </tr> <tr> <td style="padding-right: 20px;">$7 - \square = 4$</td> <td>$4 = \square - 3$</td> </tr> <tr> <td style="padding-right: 20px;">$\square - 3 = 4$</td> <td>$4 = 7 - \square$</td> </tr> <tr> <td style="padding-right: 20px;">$\square - \nabla = 4$</td> <td>$4 = \square - \nabla$</td> </tr> </table>	$7 - 3 = \square$	$\square = 7 - 3$	$7 - \square = 4$	$4 = \square - 3$	$\square - 3 = 4$	$4 = 7 - \square$	$\square - \nabla = 4$	$4 = \square - \nabla$	<p>Use of T10 where necessary</p> <p>$32 - 17$</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Subtraction for finding the difference using counting on</p> <p>e.g. $38 - 23$</p> <div style="text-align: center; margin: 10px 0;">  </div> <p><u>- = signs and missing numbers(inverse)</u></p> <p>Continue using a range of equations as in Stage 1 but with appropriate numbers.</p> <p>Extend to $14 + 5 = 20 - \square$ (inverse)</p>	
$7 - 3 = \square$	$\square = 7 - 3$									
$7 - \square = 4$	$4 = \square - 3$									
$\square - 3 = 4$	$4 = 7 - \square$									
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SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six
<p><u>Find a small difference by counting up (relating to inverse)</u> e.g. $5003 - 4996 = 7$ This can be modelled on an empty number line (see complementary addition). Children should be encouraged to use known number facts to reduce the number of steps.</p> <p><u>Use known number facts and place value to subtract</u> $92 - 25 = 67$</p>  <p><u>Counting on</u> Use of number facts to count up to find the difference (T10, T100). $754 - 568 = 186$</p>  <p><i>For those children with a secure mental image of the number line they could record the jumps only:</i></p> $\begin{array}{r} 754 \\ -568 \\ \hline 32 \text{ (600)} \\ 100 \text{ (700)} \\ \hline 54 \text{ (754)} \\ 186 \end{array}$	<p><u>Counting on</u> Use of number facts to count up to find the difference (T10, T100). This is used in the context of inverse. $14 + 168 = 182$ so: $468 - 286 = 182$</p>  <p>OR $754 - 286 = 468$</p> $\begin{array}{r} 754 \\ -286 \\ \hline 14 \text{ (300)} \\ 454 \text{ (754)} \\ \hline 468 \end{array}$ <p>Reduce the number of steps to make the calculation more efficient. <i>Extend to 2 places of decimals</i></p> <p><u>SUBTRACTION BY EXPANDED DECOMPOSITION</u> (With higher attainers secure in number facts and use of the number line).</p> <p>Subtracting with no repartitioning needed:</p> $\begin{array}{r} 345 - 123 \\ 300 + 40 + 5 \\ - (100 + 20 + 3) \\ \hline 200 + 20 + 2 \end{array}$	<p><u>Progress to 4 digit numbers</u> Teach on a number line first to subtract using T10, T100, T1000 (children should choose the most efficient method) either counting on or counting back. e.g. $8000 - 2785 = 5215$</p> <p>To make this method more efficient, the number of jumps should be reduced to a minimum through children knowing:</p> <ul style="list-style-type: none"> ▪ Complements to 1, involving decimals to two decimal places ($0.16 + 0.84$) ▪ Complements to 10, 100 and 100 <p><u>Counting on</u> $6467 - 2684 = 3783$</p>  <p>OR $6467 - 2684 = 3783$</p> $\begin{array}{r} 16 \text{ (2700)} \quad \text{can be refined to} \quad 316 \text{ (3000)} \\ 300 \text{ (3000)} \quad \quad \quad \quad \quad \quad \quad 3467 \text{ (6467)} \\ \hline 3467 \text{ (6467)} \quad \quad \quad \quad \quad \quad \quad 3783 \\ 3783 \end{array}$ <p>Reduce the number of steps to make the calculation more efficient. <i>Extend to 2 places of decimals</i></p> <p><u>Subtraction by Standard Decomposition</u></p> $\begin{array}{r} 346 - 128 \\ 3 \quad 34 \quad 16 \\ - 128 \\ \hline 218 \end{array}$

SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six
<p><u>Use known number facts and place value to subtract</u> $6.1 - 2.4 = 3.7$</p> <div style="text-align: center;"> </div> <p><u>Use known number facts and place value to subtract</u> $0.5 - 0.31 = 0.19$</p> <div style="text-align: center;"> </div> <p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>	<p>Partitioning each number and working from right to left, subtracting the bottom number from the top. Express each part as its value represented, i.e. “40 – 20”.</p> <p>Moving onto subtracting with repartition of tens only:</p> <div style="text-align: center;"> $252 - 114$ $\begin{array}{r} 200 + 50 + 2 \\ - (100 + 10 + 4) \\ \hline \end{array}$ <p style="text-align: center;">?</p> <div style="text-align: center;"> </div> $\begin{array}{r} 200 + \cancel{50} + \cancel{2} \\ - (100 + 10 + 4) \\ \hline 100 + 30 + 8 \end{array}$ </div> <p>Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4 can't be done, the next value is “REPARTITIONED”. So, “repartition 50 + 2 into 40 + 12”. It is important to cross out the whole number and replace completely. Do NOT put a ‘one in the air’! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time “12 – 4 = 8” and “40 – 10 = 30”</p> <p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>	<p><i>It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not “borrow’ a 1” and it is not “3 takeaway 1” but “300 takeaway/subtract/minus 100”).</i></p> <p><i>N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTO, decimals, etc.</i></p>

End of Year Objectives for Subtraction

Year 1 – mentally subtract O-O, TO-O, TO- TO (up to 20 e.g. 15 – 12)

Year 2 - mentally TO-O, TO-multiple of 10, mentally with informal jottings TO-TO

Year 3 – subtract mentally, HTO – O, HTO – T, HTO – H, TO - O, TO-TO. Formal written methods for TO-TO, HTO-TO, HTO-HTO

Year 4 – as above and efficient written methods for ThHTO - ThHTO, ThHTO - HTO, O.t – O.t, £O.th-£O.th

Year 5 – Efficient written methods for subtraction of 2 integers with more than 4 digits e.g. 45230 - 12432 and decimals with up to 2dp e.g. 54.34-29.78

Year 6 – as above

Please note:

There are two concepts linked to subtraction:

Subtract - where it is natural to count back to ‘take away’

Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

- Children should not move on to a written method if they are not completely confident with using a number line.
- Children will need to have had experience of different types of jumping on a number line e.g. T10 (target the ten), J10 (jump in 10s) and know how to partition numbers in different ways.
- These methods can also be easily applied, at different levels, to finding differences in values of money, measures and time.

Always present calculations horizontally in order to consider mental calculations first.

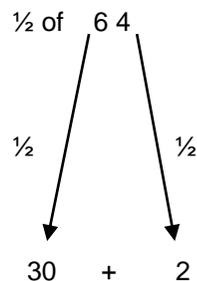
MULTIPLICATION

MENTAL STRATEGIES

Strategies to calculate the facts not yet recalled ARE essential:

$\times 2$	double	$\div 2$	halve
$\times 4$	double-double	$\div 4$	half and half again
$\times 8$	double-double-double	$\div 8$	half, half and half again

Model jottings for halving and doubling and use known facts and place value

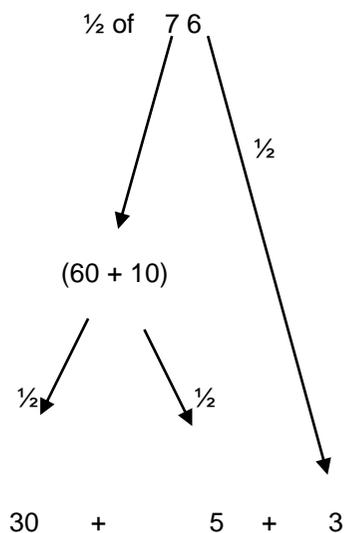


“Half of 6 tens or half of 60 is 3 tens or 30”

“Half of 4 is 2.”

Where the number of tens (or hundreds) is odd and the fact unknown, use known facts to derive the new fact:

e.g.



$\times 5$	$\frac{1}{2}$ of $\times 10$
$\times 50$	$\frac{1}{2}$ of $\times 100$
$\times 25$	$\frac{1}{4}$ of $\times 100$ (or $\frac{1}{2}$ and $\frac{1}{2}$ again of $\times 100$)
$\times 12$	$\times 10$ plus $\times 2$ (double)
$\times 15$	$\times 10$ plus $\frac{1}{2}$ of $\times 10$

MULTIPLICATION GUIDELINES

Stage One

Prerequisite skills (based on the practical)

Multiplication is related to known facts including doubling and counting groups of the same size.



$3 + 3$

E.g. use of dominoes and dice.

Counting using a variety of practical resources



Numicon and bead strings



Counting in 2s e.g. counting socks, shoes, animal's legs...

Counting in 5s e.g. counting fingers, fingers in gloves, toes...

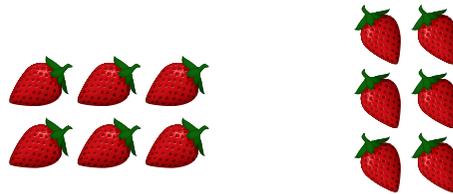
Counting in 10s e.g. fingers, toes...

Stage Two

x = signs and missing numbers

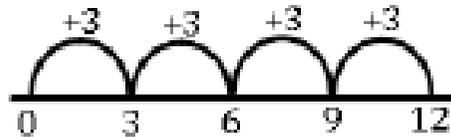
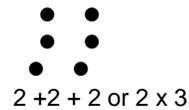
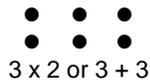
$7 \times 2 = \square$	$\square = 2 \times 7$
$7 \times \square = 14$	$14 = \square \times 7$
$\square \times 2 = 14$	$14 = 2 \times \square$
$\square \times \nabla = 14$	$14 = \square \times \nabla$

Arrays and repeated addition



Looking at rows
 $3 + 3$
2 groups of 3

Looking at rows
 $2 + 2 + 2$
3 groups of 2



If the calculation is 3×4 for example, children should understand that this means $3 + 3 + 3 + 3$. Children should also understand the commutative law and be able to use 4×3 .

Stage Three

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays and number lines (as in Stage 2).

Use known facts and place value to carry out simple multiplications

Partition

$23 \times 3 =$

X	20	3
3	$3 \times 20 =$ 60	$3 \times 3 =$ 9

Moving on to:

x	10	6
10	$10 \times 10 =$ <u>100</u>	$10 \times 6 =$ <u>60</u>
7	$7 \times 10 =$ <u>70</u>	$7 \times 6 =$ <u>42</u>

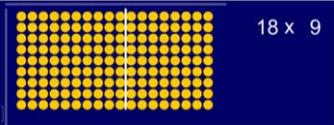
$100 + 60 + 70 + 42 = 272$

At the end of Stage 3 the children should know their 12 x 12 times tables.

MULTIPLICATION GUIDELINES

Stage One	Stage Two	Stage Three				
<p><u>Pictures / marks</u></p> <p>There are 2 socks in a pair How many socks are there in 3 pairs?</p>  <p><i>The above is required before moving on to Stage 2.</i></p>	<p><u>Partitioning</u></p> <p>Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: $6 = 5 + 1$ so e.g. Double 6 is the same as double five add double one.</p> <div style="border: 1px solid orange; padding: 5px; display: inline-block; margin-bottom: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid orange; padding: 5px 10px;">2</td> <td style="padding: 5px 10px;">3</td> </tr> </table> </div> $X 3 = 69$ <div style="border: 1px solid orange; padding: 5px; display: inline-block; margin-bottom: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid orange; padding: 5px 10px;">2</td> <td style="padding: 5px 10px;">0</td> </tr> </table> </div> $X 3 = 60$ <p>$3 \times 3 = 9$</p> <p><i>At the end of Stage 2 the children should use the above strategies, as well as doubles of multiples of 5 and knowing the 2, 3, 4, 5, 6, 8 and 10 times tables from memory.</i></p>	2	3	2	0	
2	3					
2	0					

MULTIPLICATION GUIDELINES

Stage Four	Stage Five	Stage Six																					
<p>x = signs and missing numbers Continue using a range of equations as in Stage 3 but with appropriate numbers</p> <p>Partition Continue to use arrays:</p>  <p>$18 \times 9 = 162$</p> <p>$18 \times 9 = (10 \times 9) + (8 \times 9) = 162$</p> <p>Use Multiplication array ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s.</p> <p>OR</p> <p>Use the grid method of multiplication (as below)</p> <p>$36 \times 27 =$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 10%;">x</th> <th style="width: 40%;">30</th> <th style="width: 50%;">6</th> </tr> </thead> <tbody> <tr> <td>20</td> <td>$20 \times 30 =$ <u>600</u></td> <td>$20 \times 6 =$ <u>120</u></td> </tr> <tr> <td>7</td> <td>$7 \times 30 =$ <u>210</u></td> <td>$7 \times 6 =$ <u>42</u></td> </tr> </tbody> </table> <p>$600 + 120 + 210 + 42 = 972$</p>	x	30	6	20	$20 \times 30 =$ <u>600</u>	$20 \times 6 =$ <u>120</u>	7	$7 \times 30 =$ <u>210</u>	$7 \times 6 =$ <u>42</u>	<p>Partition $47 \times 6 = 282$</p> <p>$47 \times 6 = (40 \times 6) + (7 \times 6) = 282$</p> <p>OR</p> <p>Use the grid method of multiplication (as below)</p> <p>Grid method 72×38 is approximately $70 \times 40 = 2800$</p> <p style="color: red;">Remember, always present calculations horizontally in order to consider mental calculations first.</p> <p><i>Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23×15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve $\times 2$, $\times 4$, $\times 5$, $\times 8$ which can be solved mentally using known facts.</i></p> <p>$382 \times 23 =$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 10%;">x</th> <th style="width: 25%;">300</th> <th style="width: 25%;">80</th> <th style="width: 40%;">2</th> </tr> </thead> <tbody> <tr> <td>20</td> <td>$20 \times 300 =$ <u>6000</u></td> <td>$20 \times 80 =$ <u>1600</u></td> <td>$20 \times 2 =$ <u>40</u></td> </tr> <tr> <td>3</td> <td>$3 \times 300 =$ <u>900</u></td> <td>$3 \times 80 =$ <u>240</u></td> <td>$3 \times 2 =$ <u>6</u></td> </tr> </tbody> </table> <p>$6000 + 1600 + 900 + 240 + 240 + 40 + 6 = 8986$</p> <p>$6000 + 2500 + 480 + 46 = 8000 + 980 + 46$</p> <p><i>It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process. It is also a reminder that the area of the rectangle is being calculated and the system is clear.</i></p>	x	300	80	2	20	$20 \times 300 =$ <u>6000</u>	$20 \times 80 =$ <u>1600</u>	$20 \times 2 =$ <u>40</u>	3	$3 \times 300 =$ <u>900</u>	$3 \times 80 =$ <u>240</u>	$3 \times 2 =$ <u>6</u>	<p>Use the grid method of multiplication (as below) Grid method 372×24 is approximately $400 \times 20 = 8000$</p> <p>Extend to decimals with up to two decimal places.</p> <p>The recording is reduced further, with carry digits recorded below the line.</p> $\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$ <p><i>Children who are already secure with multiplication for $TO \times O$ and $TO \times TO$ should have little difficulty in using the same method for $HTO \times TO$ or applying decimals.</i></p> <p>Long multiplication</p> <p>124×26 becomes</p> $\begin{array}{r} 1 \quad 2 \\ 1 \quad 2 \quad 4 \\ \times \quad 2 \quad 6 \\ \hline 7 \quad 4 \quad 4 \\ 2 \quad 4 \quad 8 \quad 0 \\ \hline 3 \quad 2 \quad 2 \quad 4 \\ \hline 1 \quad 1 \end{array}$ <p style="text-align: center;">Answer: 3224</p>
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MULTIPLICATION GUIDELINES

Stage Four

Stage Five

Stage Six

Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first.

Use Multiplication grid ITP to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts.

You can extend to using the grid method to multiply decimals.

Expanded Column Multiplication

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 382×23 is 'three hundreds multiplied by twenty', not 'three times two', although the relationship 3×2 should be stressed.

MULTIPLICATION GUIDELINES

Stage Four	Stage Five	Stage Six				
	<p><u>Most significant first</u></p> <p>382 x 23 =</p> <table style="width: 100%; border: none;"> <tr> <td style="text-align: right; width: 50%;"> $\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6000 \quad (20 \times 300) \\ 1600 \quad (20 \times 80) \\ 40 \quad (20 \times 2) \\ 900 \quad (3 \times 300) \\ 240 \quad (3 \times 80) \\ \hline 6 \quad (3 \times 2) \\ \hline 8786 \end{array}$ </td> <td style="text-align: right; width: 50%;"> $\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6000 \\ 1600 \\ 40 \\ 900 \\ 240 \\ \hline 6 \\ \hline 8786 \end{array}$ </td> </tr> </table> <p><u>Least significant first</u></p> <p>382 x 23 =</p> <table style="width: 100%; border: none;"> <tr> <td style="text-align: right; width: 50%;"> $\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6 \quad (3 \times 2) \\ 240 \quad (3 \times 80) \\ 900 \quad (3 \times 300) \\ 40 \quad (20 \times 2) \\ 1600 \quad (20 \times 80) \\ \hline 6000 \quad (20 \times 300) \\ \hline 8786 \end{array}$ </td> <td style="text-align: right; width: 50%;"> $\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6 \\ 240 \\ 900 \\ 40 \\ 1600 \\ \hline 6000 \\ \hline 8786 \end{array}$ </td> </tr> </table>	$\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6000 \quad (20 \times 300) \\ 1600 \quad (20 \times 80) \\ 40 \quad (20 \times 2) \\ 900 \quad (3 \times 300) \\ 240 \quad (3 \times 80) \\ \hline 6 \quad (3 \times 2) \\ \hline 8786 \end{array}$	$\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6000 \\ 1600 \\ 40 \\ 900 \\ 240 \\ \hline 6 \\ \hline 8786 \end{array}$	$\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6 \quad (3 \times 2) \\ 240 \quad (3 \times 80) \\ 900 \quad (3 \times 300) \\ 40 \quad (20 \times 2) \\ 1600 \quad (20 \times 80) \\ \hline 6000 \quad (20 \times 300) \\ \hline 8786 \end{array}$	$\begin{array}{r} 300 + 80 + 2 \\ X \quad 20 + 3 \\ \hline 6 \\ 240 \\ 900 \\ 40 \\ 1600 \\ \hline 6000 \\ \hline 8786 \end{array}$	
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End of Year Objectives for Multiplication

Year 1 – practical problems that combine groups of 2, 5 or 10

Year 2 - represent multiplication as repeated + and arrays. Practical and informal written methods and vocabulary used to support multiplication alongside known facts and mental strategies. Understand and use ‘3 for free’ for \times and \div of the 2, 3,4,5,6, 8 and 10 times-tables.

Year 3 – Describe the effect of $O \times 10$, $TO \times 10$, $O \times 100$, $TO \times 100$. Practical and informal written methods for $TO \times O$.

Year 4 – Derive and recall \times and \div facts up to 12×12 and ‘3 for free’ facts. Multiply numbers to 1000 by 10 and 100. Formal written layout and explain $TO/HTO \times O$.

Year 5 – mentally multiply $TO \times O$. Multiply whole numbers and decimals by 10, 100 and 1000. Formal written methods to multiply $ThHTO \times O$, $ThHTO \times TO$, $O.t \times O$

Year 6 – mentally calculate $TO \times O$, $O.t \times O$. Formal written methods to multiply up to 4 digit by 2 digit and one digit with up to 2 decimal places.

As with addition and subtraction, before progressing through the stages of calculation:

Learning

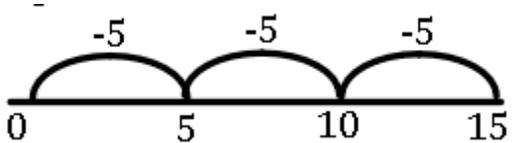
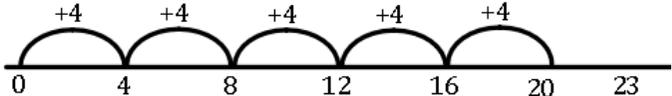
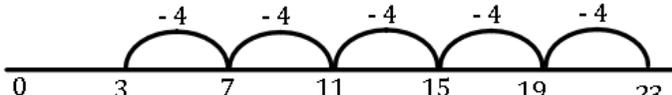
- It is crucial to know or be able to derive key number facts:
 - ⇒ Understand and use doubling and halving
 - ⇒ $\times/\div 10$ (as moving a place to the left/right NOT “add a zero” etc.!!)
- Place value and partitioning **MUST** be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23×15 should not be calculated vertically. Consider use of numbers carefully.

Always present calculations horizontally in order to consider mental calculations first.

DIVISION GUIDELINES

Stage One	Stage Two	Stage Three
<p>Prerequisite skills (based on the practical) Understanding the language of half in different contexts. Know halves of even numbers up to 10.</p> <p>Sharing Requires secure counting skills -see counting and understanding number strand Develops importance of one-to-one correspondence See appendix for additional information on x and ÷ and aspects of number</p> <p>Sharing – 6 sweets are shared between 2 people. How many do they have each?</p> <div style="text-align: center;">  </div> <p>Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.</p> <p>Grouping Sorting objects into 2s / 5s/ 10s etc. How many pairs of socks are there?</p> <div style="text-align: center;">  </div> <p>There are 10 bulbs. Plant 5 in each pot. How many pots are there? Jo has 10 Lego wheels. How many bicycles can she make?</p>	<p>÷ = signs and missing numbers $6 \div 2 = \square$ $\square = 6 \div 2$ $6 \div \square = 3$ $3 = 6 \div \square$ $\square \div 2 = 3$ $3 = \square \div 2$ $\square \div \nabla = 3$ $3 = \square \div \nabla$</p> <p>Grouping Link to counting and understanding number strand Count up to 100 objects by grouping them and counting in tens, fives or twos;... Find one half, one quarter and three quarters of shapes and sets of objects $15 \div 5$ can be modelled as: There are 15 strawberries. How many people can have 5 each? How many 5s make 15?</p> <p>$15 \div 5$ can be modelled as repeated subtraction</p> <div style="text-align: center;">  </div> <p>In the context of money count forwards and backwards using 2p, 5p and 10p coins</p> <p>Practical grouping e.g. in PE</p> <p>12 children get into teams of 4 to play a game. How many teams are there?</p> <div style="text-align: center;">  </div> <p>Children should know that division is not commutative.</p>	<p>÷ = signs and missing numbers Continue using a range of equations as in Stage 2 but with appropriate numbers.</p> <p>Understand division as sharing and grouping $24 \div 3$ can be modelled as: Sharing – 24 shared between 3</p> <p>OR Grouping - How many 3's make 24?</p> <p>Remainders $23 \div 4 = 5r3$ Sharing - 23 shared between 4, how many left over? Grouping – How many 4's make 23, how many left over? e.g.</p> <div style="text-align: center;">   </div>

DIVISION GUIDELINES

Stage Four	Stage Five	Stage Six																																						
<p><u>÷ = signs and missing numbers</u></p> <p>Continue using a range of equations as in Stage 2 but with appropriate numbers.</p> <p><u>Sharing and grouping</u></p> <p>60 ÷ 12 can be modelled as:</p> <p>grouping – 12 subtracted repeatedly from 60 on a no. line, leading to subtracting ‘groups’ of 12.</p> <p>sharing – sharing among 12, the number given to each person.</p> <p><u>Remainders</u></p> <p>41 ÷ 4 = 10 r1</p>  <p>41 = (10 × 4) + 1</p> <p><u>Pencil and paper procedures- Chunking.</u></p> <p>72 ÷ 5 lies between 50 ÷ 5 = 10 and 100 ÷ 5 = 20</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>Key Facts</p> <p>1 × 5 = 5</p> <p>2 × 5 = 10</p> <p>5 × 5 = 25</p> <p>10 × 5 = 50</p> </div> <table style="margin-left: 100px;"> <tr><td style="text-align: right;">72</td><td></td></tr> <tr><td style="text-align: right;">- 50</td><td>(10 groups)</td></tr> <tr><td style="text-align: right;">22</td><td></td></tr> <tr><td style="text-align: right;">- 20</td><td>(4 groups)</td></tr> <tr><td style="text-align: right;">2</td><td></td></tr> </table> <p style="text-align: right;">Answer : 14 remainder 2</p>	72		- 50	(10 groups)	22		- 20	(4 groups)	2		<p><u>Sharing and grouping</u></p> <p>Continue to understand division as both sharing and grouping (repeated subtraction).</p> <p><u>Remainders</u></p> <p><u>Pencil and paper procedures- Chunking</u></p> <p>256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40</p> <table style="margin-left: 100px;"> <tr><td style="text-align: right;">256</td><td></td></tr> <tr><td style="text-align: right;">- 210</td><td>7 × 30</td></tr> <tr><td style="text-align: right;">46</td><td></td></tr> <tr><td style="text-align: right;">- 42</td><td>7 × 6</td></tr> <tr><td style="text-align: right;">4</td><td></td></tr> </table> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>Key Facts</p> <p>1 × 7 = 7</p> <p>2 × 7 = 14</p> <p>5 × 7 = 35</p> <p>10 × 7 = 70</p> </div> <p style="text-align: right;">Answer: 36 remainder 4</p> <p>Quotients expressed as fractions or decimal fractions</p> <p>61 ÷ 4 = 15 ¼ or 15.25</p> <p><u>Also, Short Division for More Able Children</u></p> <p>432 ÷ 5 becomes</p> <table style="margin-left: 100px;"> <tr><td style="text-align: right;">8 6 r2</td></tr> <tr><td style="text-align: right;">5 4 3 2</td></tr> </table> <p style="text-align: right;">Answer: 86 remainder 2</p> <p>Considering each column starting from the left.</p>	256		- 210	7 × 30	46		- 42	7 × 6	4		8 6 r2	5 4 3 2	<p><u>Sharing, grouping and remainders as Stage Five</u></p> <p><u>Pencil and paper procedures- Chunking</u></p> <p>977 ÷ 36 is approximately 1000 ÷ 40 =</p> <table style="margin-left: 100px;"> <tr><td style="text-align: right;">977</td><td></td></tr> <tr><td style="text-align: right;">- 720</td><td>36 × 20</td></tr> <tr><td style="text-align: right;">257</td><td></td></tr> <tr><td style="text-align: right;">- 180</td><td>36 × 5</td></tr> <tr><td style="text-align: right;">77</td><td></td></tr> <tr><td style="text-align: right;">- 72</td><td>36 × 2</td></tr> <tr><td style="text-align: right;">5</td><td></td></tr> </table> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>Key Facts</p> <p>1 × 36 = 36</p> <p>2 × 36 = 72</p> <p>5 × 36 = 180</p> <p>10 × 36 = 360</p> </div> <p style="text-align: right;">Answer: 27 5/36</p> <p><u>Pencil and Paper procedures- Short Division Method</u></p> <p style="text-align: center;"> divisor 5 847 dividend </p> <p style="text-align: center; margin-top: 20px;">496 ÷ 11 becomes</p> <table style="margin-left: 100px;"> <tr><td style="text-align: right;">4 5 r1</td></tr> <tr><td style="text-align: right;">1 1 4 9 6</td></tr> </table> <p style="text-align: right;">Answer: 45 1/11</p> <p>Both methods above are necessary at this stage, to deal with the wide range of problems experienced at Stage Six.</p>	977		- 720	36 × 20	257		- 180	36 × 5	77		- 72	36 × 2	5		4 5 r1	1 1 4 9 6
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End of Year Objectives for Division

Year 1 – practical problems that share into equal groups of 2, 5 or 10.

Year 2 – derive and recall division facts for 2, 5 or 10, represent division as repeated subtraction (grouping) and sharing.

Practical and informal written methods and vocabulary used to support division, including remainders. To know that division is not commutative.

Year 3 – Practical and informal written methods for $TO \div O$. Understand and use ‘3 for free’ for \times and \div of the 2, 3, 4, 5, 6, 8 and 10 times-tables. Round remainders up or down, depending on the context.

Year 4 – Derive and recall \times facts up to 12×12 and apply ‘3 for free’ facts. Divide numbers to 1000 by 10 and 100. Develop and use formal written layouts to record.

Year 5 – Divide whole numbers and decimals by 10, 100 and 1000. Divide numbers up to 4 digits by a one digit number using the formal written methods for division and interpret remainders appropriately for the context.

Year 6 – Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division interpreting remainders as fractions, decimals, etc. Divide numbers up to 4 digits by a two digit number using the formal written methods for division and interpret remainders appropriately for the context.

As with multiplication, before progressing through the stages of calculation:

Learning

- It is crucial to know or be able to derive key number facts:
 - ⇒ Understand and use doubling and halving
 - ⇒ $\times/\div 10$ (as moving a place to the left/right NOT “add a zero” etc.!!)
- Place value and partitioning **MUST** be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. $24 \div 3$ should not be calculated using short division. Consider use of numbers carefully.